

Study comparative in two control design methods for stabilizing discrete time

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Abstract— The two control design methods for stabilizing discrete time systems are based on the notion of exact linearization with diffeomorphism and feedback. On the one hand, the first method consists in developing a polynomial control law based on the exact linearization approach. This approach was developed by considering the concept of relative degree. On the other hand, the second method is based on the methodology of control by state-return for which there is a linearizing transformation of the looped system.

These approaches are based on the reversing trajectory method in order to estimate the asymptotic stability region around an operating point. The effectiveness of the suggested methods is tested by numerical examples of the CSTR chemical reactor described by a strongly non-linear model.

Keywords— Input-output feedback linearization, Polynomial Nonlinear System, Asymptotic Stability Region, Reversing Trajectory Method, Discrete-Time.

I. INTRODUCTION

The theory of analysis and synthesis of the dynamic systems has seen significant developments during the last years [1]-[2]. These developments are related to various aspects of the automatic control and, in particular, the nonlinear control. In this communication, we are interested in a comparative study of two nonlinear control approaches.

The first approach is developed in the context of differential geometry. This method aims to transform the original nonlinear system into an exact linear equivalent model via a state feedback and a coordinate transformation [3]-[4]. In fact, the formalism of exact feedback linearization is growing in popularity.

This approach was developed by considering the concept of relative degree r .

The relative degree is of an increasing importance to define the strategies of possible control for the nonlinear process [5]. In fact, when the relative degree is inferior to the order of the

studied system; it is possible to establish a dynamic compensator that can be bounded or unbounded.

When it is bounded, the variables of the compensator taking part in the characterization of diffeomorphism and feedback may help to ensure a control for the trajectory tracking [3]. On the other side of the coin, the objective of the control is too ambitious. In fact, all that one can do is a simple regulation in the neighborhood of an operating point.

The second method is based on the methodology of control by state-return for which there is a linearizing transformation of the looped system. However, it proves to be more advantageous than the first control in the case of regulation in minimum time [6].

However, the control synthesis in the discrete-time domains requires powerful analytical tools especially for the class of nonlinear systems [3], [5]-[7]. It is intended to develop an original strategy to study and analyze the control of discrete time nonlinear systems.

This communication is organized in the following way: the approach of exact linearizing control is presented in the first part. The second part is devoted to the study and the synthesis of the analytical control approach. Moreover, the design methodology takes advantage of the Reversing Trajectory Method (RTM) in order to maximize the domain of attraction around the operating point [8].

The last part is dedicated to the validation by simulation of the proposed comparative study which is applied it to a nonlinear model of a CSTR chemical reactor [9].

II. THE EXACT LINEARIZING CONTROL APPROACH

A. Theoretical formalism of the exact linearizing control

If we consider the refined mono-variable nonlinear system given by the following equation system:

$$\begin{cases} X_{k+1} = F(X_k) + G(X_k)U_k \\ y = h(X_k) \end{cases} \quad (1)$$

where $k = 0, 1, \dots$ is the discrete-time index, $X_k \in \mathfrak{R}^n$ is the vector of state variables, $U_k \in \mathfrak{R}^m$ is the input variable and $y \in \mathfrak{R}$ is the output variable. The field of $F(\cdot)$ and $G(\cdot)$ are supposed to be nonlinear of an unknown analytical form, $h(\cdot)$ is also an unknown nonlinear analytical function.

Definition:

For system (1), let us denote by \circ the usual composition of the function and, recursively, define the following functions:

$$\begin{cases} h^0(X_k) = h(X_k) \\ h^k(X_k, U_k) = h^{k-1} \circ F(X_k) \end{cases} \quad (2)$$

with $k \geq 1$. The relative degree r of system (1) in the neighborhood of X_0 is defined as the smallest integer for which:

$$\left. \frac{\partial}{\partial U_k} h^r(X_k, U_k) \right|_{X_{k,0}} \neq 0 \quad (3)$$

on $(\mathfrak{R}^n \times \mathfrak{R})$. The relative degree r determines the time delay of the input signal U before it can influence the system output y .

The problem in linearization by looping is to find smooth functions $q(\cdot)$ and $p(\cdot)$ with $q(X_{k,0}) \neq 0$ is a diffeomorphism T with $T(X_{k,0}) = 0$.

as we are given the mono-variable nonlinear system (1) of relative degree r in $X_{k,0}$ [3]-[4], we may pose :

$$\begin{cases} T_1(X_k) = h(X_k) \\ T_2(X_k) = L_F h(X_k) \\ \vdots \\ T_r(X_k) = L_F^{r-1} h(X_k) \end{cases} \quad (4)$$

When the relative degree is $r < n$ in $X_{k,0}$, one can put the system in a particular form that it is called the normal form, it is then possible to find $(n-r)$ functions $T_{r+1}(X_k) \dots T_n(X_k)$ where the value of the functions in $X_{k,0}$ can be chosen as:

$$\left\langle \frac{\partial T_i(X_k)}{\partial X_k}, G(X_k) \right\rangle = L_G T_i(X_k) = 0 \quad (5)$$

for $r+1 \leq i \leq n$

$L_G T_i(X_k)$ indicates the derivative of Lie of the function $T_i(X_k)$ compared with $G(X_k)$.

The expression of diffeomorphism $T(X_k)$ can be written then as:

$$T(X_k) = \begin{bmatrix} h(X_k) \\ L_F h(X_k) \\ \vdots \\ L_F^r h(X_k) \\ \xi_i(X_k) \\ \vdots \\ \xi_{n-r}(X_k) \end{bmatrix} \quad (6)$$

The expression of the dynamic compensator $\xi_i(X_k)$ of $(n-r)$ components is determined by solving equation (5).

The transformed variables: $Z_k = T(X_k)$, the resulting control system by a linear system, are equivalent to Brunovsky of the form:

$$Z_{k+1} = M(X_k)Z_k + N(X_k)v_k \quad (7)$$

Where the pair (M, N) is controllable. The new state Z_k is called linearizing state and the control law v_k is a linearizing control law and, thus, we have:

$$v_k = p(X_k) + q(X_k)U_k \quad (8)$$

where also:

$$U_k = -\frac{p(X_k)}{q(X_k)} + \frac{v_k}{q(X_k)} = \alpha(X_k) + \beta(X_k) \quad (8)$$

B. Polynomial approach of the synthesis of the exact linearizing control

When the variables of the dynamic compensator synthesized by equation (5) are not bounded, it would be possible only to ensure a regulation around the desired operation. The trajectory tracking is henceforth an unrealizable objective using a feedback built around the variables of the compensator.

The objective of this paragraph is to develop an analytical technique allowing for the synthesis of the linearizing feedback when the relative degree r is strictly lower than order n of the system and when the states of the dynamic compensator are not bounded.

1) *Synthesis of the variation polynomial model:*

Let us consider the following variable change:

$$\begin{cases} x_k = X_k - X_{k,n} \\ u_k = U_k - U_{k,n} \end{cases} \quad (9)$$

where $X_{k,n}$ the state vector in an operating point and $U_{k,n}$ is the control.

The model of nonlinear variation polynomial model is expressed by the following equation:

$$x_{k+1} = \sum_{i \geq 1} f_i x_k^{[i]} + \sum_{j \geq 0} g_j (I_m \otimes x_k^{[j]}) u_k \quad (10)$$

where:

x_k : is the variation state vector

u_k : is the control vector

2) Characterization of the feedback dynamic

When writing in the form of a polynomial development of the dynamic feedback u which meets the needs of the linearizing exact input-output control, we will get:

$$u_k = -\frac{L_f^r h(x_k)}{L_g L_f^{r-1} h(x_k)} + \frac{1}{L_g L_f^{r-1} h(x_k)} v_k \quad (11)$$

Where we consider that the quantities $L_f^r h(x_k)$ and $L_g L_f^{r-1} h(x_k)$ are expressed by the following quantities:

$$\begin{cases} L_f^r h(x_k) = \sum_{i=1}^r L_i x_k^{[i]} \\ L_g L_f^{r-1} h(x_k) = \sum_{j=0}^{r-1} J_j x_k^{[j]} \end{cases} \quad (12)$$

The control v can be determined by a simple placement of the poles. It becomes then:

$$v_k = -KT(x_k) - K \sum_{s=1}^r T_s x_k^{[s]} \quad (13)$$

where $K = [k_1 \ k_2 \ \dots \ k_n]$ is the gain vector of the control which is determined by the placement of the poles.

The synthesis of the polynomial writing of the feedback which is obtained by the combination of the various equations lets us express in the following form:

$$u_k = \sum_{i=1}^r \chi_i x_k^{[i]} + \sum_{j=0}^{r-1} \mu_j x_k^{[j]} \quad (14)$$

with:

$$\begin{cases} \chi_1 = (J_0^{-1}) L_1 \\ \vdots \\ \chi_i = J_0^{-1} \left[L_i - \sum_{p=1}^{i-1} (J_{i-p} \otimes \chi_p) \right] \quad i \geq 1 \end{cases}$$

and

$$\begin{cases} \mu_1 = (J_0^{-1}) K T_1 \\ \vdots \\ \mu_j = J_0^{-1} \left[K T_j - \sum_{p=0}^{j-1} (J_{j-p} \otimes \mu_p) \right] \quad j \geq 1 \end{cases}$$

By replacing u_k with its polynomial expression defined by system (14) in the equation (10), we easily obtain the autonomous form of this model which is expressed under the form:

$$x_{k+1} = \sum_{i=1}^r P_i x_k^{[i]} \quad (15)$$

Where:

$$\begin{cases} P_1 = f_1 + g_0 (\chi_1 + \mu_1) \\ \vdots \\ P_i = f_i + \sum_{j=0}^i (g_j \otimes \chi_{i-j}) + \sum_{j=0}^i (g_j \otimes \mu_{i-j}) \quad i \geq 1 \end{cases}$$

III. THE APPROACH OF THE ANALYTICAL CONTROL

Let us consider the following variable change defined in equation (9). Using the development into generalized expansion Taylor series and Kronecker tensorial product, the model (1) can be readily transformed into a variation polynomial model expressed by the equation (10).

Such a model is, after that, used to characterize the feedback linearizing control which ensures a regulation around an operating point and which shall vary along a desired trajectory. It is easy to express the feedback in the following polynomial form:

$$u_k = \sum_{i=1}^q \lambda_i x_k^{[i]} \quad (16)$$

and a nonlinear analytic transformation [10]-[11]:

$$z_{k+1} = Dz_k = \Psi(x_k) \quad (17)$$

The design methodology consists in the search of an attraction domain around an operating point.

This problem requires the setting into the autonomous form of the state equation (16)

By rearranging the equations (10) and (16), we obtain the autonomous form given by:

$$x_{k+1} = \sum_{i=1}^q A_i x_k^{[i]} \quad (18)$$

where $A_i = f_i + \sum_{j=0}^{i-1} g_j (\lambda_{i-j} \otimes I_{n_j})$

The equation defined by the analytical transformation proves that:

$$z_{k+1} = \sum_{i \geq 1} \Psi_i x_k^{[i]} \quad (19)$$

The relation (19) truncated to an order q is noted as:

$$z_{k+1} = \Psi_1 \sum_{i \geq 1} f_i x_k^{[i]} + \Psi_2 \sum_{i \geq 2} f_i x_k^{[i]} + \dots + \Psi_q \sum_{i \geq q} f_i x_k^{[i]} \quad (20)$$

where also

$$z_{k+1} = \Psi_1 f_1^1 R_n^1 \tilde{x}_k + \dots + (\Psi_1 f_q^1 R_n^q + \dots + \Psi_q f_q^q R_n^q) \tilde{x}_k^{[q]} \quad (21)$$

In addition, by replacing the vector z_k in (17) with its development given by (19), it becomes:

$$z_{k+1} = D z_k = \sum_{i \geq 1} D \Psi_i R_n^i \tilde{x}_k^{[i]} \quad (22)$$

After identifying the relations (21), (22) and by introducing the function 'vec' into the Sylvester equation, it comes

$$A_i \text{vec}(\Psi_i) = B_i + C_i \text{vec}(\lambda_i) \quad (23)$$

with

$$A_i = \left[\left((R_n^i)^T \otimes D \right) - \left((f_i + g_0 \lambda_i)^{[i]} R_n^i \right)^T \otimes I_n \right]$$

$$B_i = \text{vec} \left(\begin{array}{l} f_i + \sum_{j=1}^{i-1} g_j (\lambda_{i-j} \otimes I_{n_j}) \\ \dots + \Psi_{i-1} f_{i-1} R_n^{i-1} \end{array} \right) R_n^i + \Psi_2 f_i^2 R_n^i + \dots$$

$$C_i = \left((R_n^i)^T \otimes g_0 \right)$$

the relation (23) leads to:

$$\text{vec}(\Psi_i) = -A_i^+ B_i + A_i^+ C_i \text{vec}(\lambda_i) \quad (24)$$

and

$$\text{vec}(\lambda_i) = -\left(A_i^+ C_i \right)^+ A_i^+ B_i \quad (25)$$

where A_i^+ is the pseudo-inverse of A_i

The readers can refer to [11] for more details.

IV. THE DISCRETE REVERSING TRAJECTORY METHOD

The discrete RTM is theoretically exploitable for any locally stable nonlinear system. Moreover, it proves its effectiveness through the advantage of being numerically implementable for high nature systems. What is more, one may also note that the performance of this method largely depends on the determination of an asymptotically small initial area which will be used as an initial field for integration in opposite direction [8], [12]-[13]. This approach considers an initial field of asymptotic stability around an equilibrium point to execute, thereafter, reversing iterations allowing widening the initial field of the considered stability.

Let us consider the recurrent autonomous state equation expressed by:

$$x_{k+1} = S(x_k) = \sum_{i=1}^q S_i x_k^{[i]} \quad (26)$$

where $S_i, i = 1, \dots, q$ are the dimension matrices $(n \times n^i)$, q a truncation order defined before $x_k^{[i]}$ indicates the Kronecker tensorial power of order i of the state vector x_0 such that:

$$\lim_{k \rightarrow \infty} x(k, x_0) = 0 \quad (27)$$

Such an area will be noted by Ω and characterized by a surface Γ .

In the following part, we will present a theorem describing the application of the RTM to the case of discrete-time nonlinear systems.

The method that we use to determine the attraction domain of radius R_0 is based on the following theorem [14]:

Theorem: ASR Ω is an invariant and an open unit.

We take into consideration the discrete nonlinear system defined by the following recurrent state equation:

$$x_{k+1} = S_1 X_k + f(k, x_k) \quad (28)$$

We suppose that the linear part of the model defined by equation (10) is asymptotically stable, i.e. the matrix S_i is of Schur and verifies the inequality:

$$\|S_1^{k-k_0}\| \leq c \alpha^{k-k_0} \quad \forall k \geq k_0 \quad (29)$$

c and $\alpha \in \mathfrak{R}^+$

The discrete time state variables of equation (28) are exponentially stable in the ball $B(o, R_0)$.

where R_0 is the unique positive solution of the following equation [11]:

$$\sum_{k=2}^q c^{k-1} \|S_k\| R_0^{k-1} - \frac{1-\alpha}{c} = 0 \quad (30)$$

It may be expressed in the following form:

$$x_k = S^{-1}(x_{k+1}) = \pi(x_{k+1}) \quad (31)$$

where $\pi(\cdot)$ is a polynomial vector function which can be developed into generalized Taylor series. Then, we have

$$x_k = \sum_{i=1}^q \pi_i x_{k+1}^{[i]} \quad (32)$$

where $\pi_i, i=1,2,\dots,q$ are matrices of dimensions $(n \times n^{[i]})$.

By identifying the matrices π_i in Eq. (32) and by generating the relation $S(\pi(x_{k+1})) = x_{k+1}$, we obtain the following recurrent relations:

$$\begin{cases} \pi_1 = \pi_1^1 = \pi_1^1 \\ \pi_2 = \pi_2^1 = -\pi_1^{-1} \pi_2^1 \\ \vdots \\ \pi_q = \pi_q^1 = -S_1^{-1} \left(\sum_{i=1}^{q-1} S_i \pi_i^1 \right) \end{cases} \quad (33)$$

Where the matrices π_p^i , for $i=2,\dots,q$ and $p=2,\dots,q$ are given by:

$$\begin{cases} \pi_p^2 = \sum_{j=1}^{p-1} (\pi_{p-j}^1 \otimes \pi_j^1) \\ \vdots \\ \pi_p^i = \sum_{j=1}^{p-i+1} (\pi_{p-j}^{i-1} \otimes \pi_j^1) \end{cases} \quad (34)$$

The asymptotic stability region estimation of the system Eq. (26) using the RTM involves six steps which are described in details in [15] - [16]:

V. APPLICATION OF THE COMPARATIVE STUDY TO A CSTR REACTOR

As it has already been mentioned, the objective of this communication is a comparative study between two control approaches of a CSTR chemical reactor.

A. Representation of the state

We study, in this section, a nonlinear model of the shown irreversible chemical reactor. The process model can be written as:

$$\begin{cases} \frac{dC_A}{dt} = \frac{Q_f}{V_r} (C_{Af} - C_A) - k_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \frac{dT}{dt} = \frac{Q_f}{V_r} (T_f - T) + \frac{(-\Delta H)}{\rho C_p} k_0 \exp\left(-\frac{E}{RT}\right) C_A + \frac{U_A}{\rho C_p V_r} (T_j - T) \end{cases} \quad (36)$$

Where $C_A [mol/m^3]$ is the concentration of the A component in the reactor, $T [K]$ is the reactor temperature, while the input of the process is T_j : the temperature of the reactor jacket.

The considered controlled output y of the process is the reactor temperature T . The parameters and their nominal values are given in [17]. Let us consider the following notations: $X_1 = C_A$, $X_2 = T$, $U = T_j$ and $y = X_2$ is the process output.

The model of the reactor can be written in the following form:

$$\dot{X} = E_1 X + E_2 X^{[2]} + E_3 X^{[3]} + D_0 U \quad (37)$$

By considering a suitable selected discretization period T and by adopting the equation in the following approximations in an interval $[kT, (k+1)T]$, [18]:

$$\begin{aligned} X(t) &= \frac{X_{k+1} + X_k}{2}, \\ \dot{X}(t) &= \frac{X_{k+1} - X_k}{T}, \\ U(t) &= U_k. \end{aligned} \quad (38)$$

the following discrete model defined in the equation truncated to the second order can be derived:

$$X_{k+1} = F_1 X_k + F_2 X_k^{[2]} + G_0 U_k \quad (39)$$

where

$$F_1 = \left(\frac{I_n - E_1}{T} - \frac{E_1}{2} \right)^{-1} \left(\frac{I_n + E_1}{T} + \frac{E_1}{2} \right), \quad F_2 = \left(\frac{I_n - E_1}{T} - \frac{E_1}{2} \right)^{-1} E_2,$$

$$F_3 = \left(\frac{I_n - E_1}{T} - \frac{E_1}{2} \right)^{-1} E_3, \quad G_0 = \left(\frac{I_n - E_1}{T} - \frac{E_1}{2} \right)^{-1} D_0.$$

The discretization of the reactor model leads to a polynomial model of the form (39), where:

$$\begin{aligned} F_1 &= \begin{bmatrix} 0.9048 & 0 \\ 0 & 0.7322 \end{bmatrix} \\ F_2 &= \begin{bmatrix} 0 & -0.042 & -0.042 & -0.0011 \\ 0 & 0.807 & 0.807 & 0.2138 \end{bmatrix} \end{aligned}$$

$$F_3 = \begin{bmatrix} 0 & -0.0037 & 0 & -0.0037 & -0.0037 & -0.0009 \\ 3 \times 2 & 0.7126 & 0 & 0.7126 & 0.7126 & 0.1806 \end{bmatrix}$$

$$G_0 = \begin{bmatrix} 0 \\ 0.1812 \end{bmatrix}$$

The polynomial development (truncated to the third order) of the reactor model around an operating point $(X_{k,n}, U_{k,n})$ entails the following nonlinear variation model:

$$x_{k+1} = f_1 X_k + f_2 x_k^{[2]} + g_0 u_k \quad (40)$$

B. Study of the first approach

The relative degree of the system is, thus, equal to 1 which is strictly lower than the system control. The dynamics of the system (15) is so decomposed into an input-output part and an unobservable internal one. By using the change of coordinates (6), the model of the reactor will be transformed into the normal form which is written as:

$$T(X_k) = (X_{2,k} \xi_1(X_k))^t \quad (41)$$

An expression of the dynamic compensator is determined by solving the equation (5). A possible solution for this problem is translated by the equation (33) which spells:

$$\xi_1(X_k) = X_{1,k} X_{2,k} \quad (42)$$

As it was already mentioned, the objective of this work is to ensure the tracking trajectory by the reaction temperature of the chemical reactor.

The expression of diffeomorphism is described by the equation (42) which is developed under a polynomial form truncated to the third order:

$$T(x_k) = T_1 x_k + T_2 x_k^{[2]} + T_3 x_k^{[3]} \quad (43)$$

The control is also expressed in the following form:

$$u_k = \alpha_1 x_k + \alpha_2 x_k^{[2]} + \alpha_3 x_k^{[3]} + \mu_1 x_k + \mu_2 x_k^{[2]} + \mu_3 x_k^{[3]} \quad (44)$$

C. Study of the second approach

The problem of control is meant to determine the feedback law. Hence, it is easy to express the nonlinear state feedback controls law in this polynomial form:

$$u_k = \lambda_1 x_k + \lambda_2 x_k^{[2]} + \lambda_3 x_k^{[3]} \quad (45)$$

The system feedback is described by the linear system

$$z_{k+1} = D z_k \quad (46)$$

where z_k is used to express the polynomial form

$$z_k = x_k + \Psi_2 x_k^{[2]} + \Psi_3 x_k^{[3]} \quad (47)$$

The application of the suggested approach leads to:

The matrix λ_1 is selected providing that the dynamics of the linear system defined by a double pole in the module is equal to 0.63. This yields $\lambda_1 = [-0.1 \quad 0.16]$.

The matrices Ψ_2 and Ψ_3 and the Co-vectors λ_2 and λ_3 are determined by using the relations (24) and (25) are:

$$\Psi_2 = \begin{bmatrix} -0.3086 & -0.8252 & -0.0493 & 0.1827 \\ -0.0928 & 0.1044 & -0.0593 & 0.0952 \end{bmatrix}$$

$$\Psi_3 = \begin{bmatrix} -0.0190 & -0.0835 & -0.0196 & -0.0458 & -0.0018 & 0.0068 & -0.0047 & 0.0081 \\ -0.0169 & -0.0458 & -0.0096 & 0.0025 & -0.0047 & -0.0047 & 0.0093 & 0.0015 \end{bmatrix}$$

$$\lambda_2 = [-0.1631 \quad -0.0543 \quad -0.0878 \quad -0.0335]$$

$$\lambda_3 = [-1.2924 \quad 0.2590 \quad -0.0426 \quad -0.0317 \quad 0.2590 \quad -0.0693 \quad -0.0317 \quad -0.0325]$$

D. Study of the second approach

This study is developed via the exploitation of Matlab Toolbox control.

The polynomial input-output linearizing control defined by the equations (35) and (36) was implemented in order to ensure the regulation of the process around the operating point $(X_{k,1n} = 0.98 \text{ mol/m}^3, X_{k,2n} = 320 \text{ K})$.

The simulation results are given by the figures 1, 2 and 3. On these figures, there is a dynamic behavior characterized by satisfactory performances regarding the regulation of the controlled variables. Indeed, we may notice that the simulated variation variables quickly go back to the origin (in a few seconds) and that the recorded excesses remain within the tolerated limits. We note that the study of the variation variable by the analytical approach returns to the origin more quickly than the variation variable by the exact linearization approach.

Besides, the control signal, which is represented by figure 3, is characterized by a completely acceptable variation in the process.

In figures 4 and 5, we show the ASR which is obtained by means of the reverse trajectory approach. The initial conditions for integrating the reverse autonomous polynomial system are taken on the circular bound of the Initial Asymptotic Stability Region around an operating point applied to these two methods.

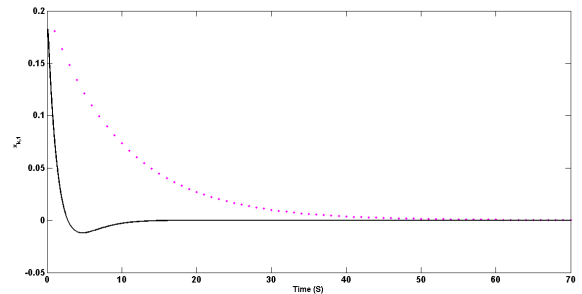


Fig. 1 Evolution of the variation variable $x_{k,1}$

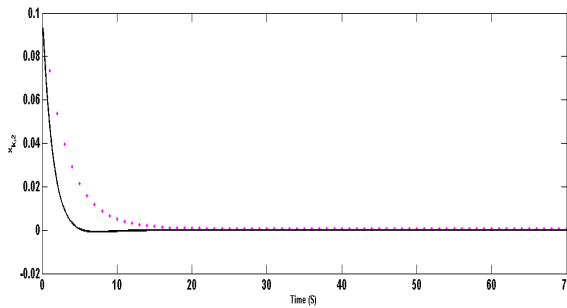


Fig. 2 Evolution of the variation variable $x_{k,2}$

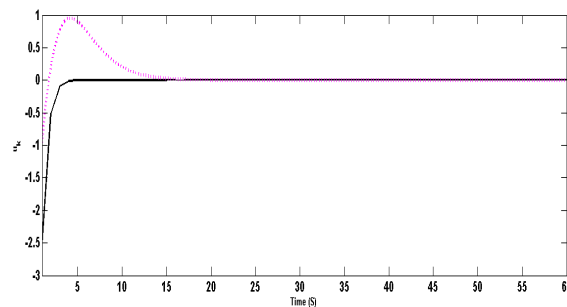


Fig. 3 Evolution of the control variable

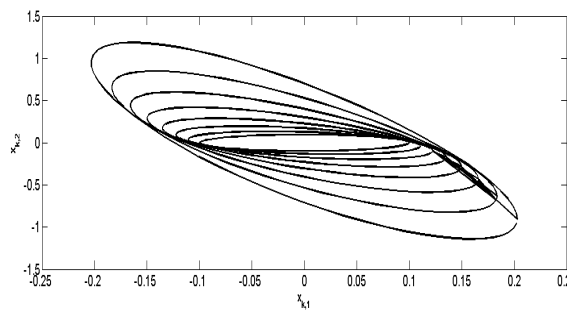


Fig. 4 Enlargement of the ASR using the RTM of the first approach

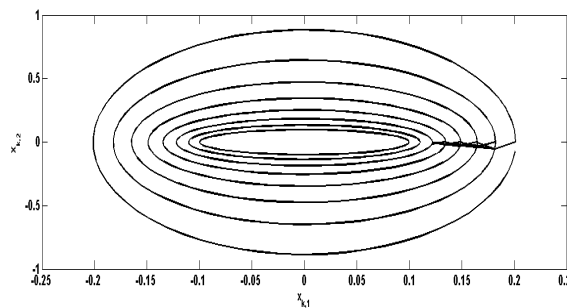


Fig. 5 Enlargement of the ASR using the RTM of the second approach



VI. CONCLUSION

A comparative study is conducted between the exact linearization control and the analytical control.

The first is based on the concept of relative degree and the second is based on the formalism of the linearization feedback.

These approaches are mainly based on the use of the expansion Taylor series and the Kronecker tensorial power. The advantage of the analytic approach can be summarized in the fact that one can analyze the nonlinear discrete control problem in a generic context. Furthermore, the pre-specified performances are studied based on the reference linear model. The asymptotic stability region in the neighborhood of the operating point is larger than the region obtained by the local linearized model. This is a powerful characteristic of the proposed approach since it can be implemented to gain a scheduling technique without requiring many operating points in the tracked trajectory.

The implementation of the suggested technique to the control of a CSTR reactor showed a perfect performance. This technique can ensure a sound control of the nonlinear process

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